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Generalized Magneto-thermoelasticity in a Fiber-Reinforced Anisotropic Half-Space

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Abstract The propagation of plane waves in a fiber-reinforced, anisotropic thermoelastic half-space proposed by Lord–Shulman under the effect of a magnetic field is discussed. The problem has been solved numerically using a finite element method. Numerical results for the temperature distribution, the displacement components, and the thermal stress are given and illustrated graphically. Comparisons are made with the results predicted by the theory of generalized thermoelasticity with one relaxation time for different values of time. It is found that the reinforcement has a great effect on the distribution of field quantities.

Keywords Anisotropic material · Fiber-reinforced · Finite element method · Generalized magneto-thermoelasticity

1 Introduction

Lord and Shulman [1] introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. The theory was extended for an anisotropic

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body by Dhaliwal and Sherief [2]. In this theory, a modified law of heat conduction including both the heat flux and its time derivatives replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both coupled and uncoupled theories of thermoelasticity. Erdem [3] derived the heat conduction equation for a composite rigid material containing an arbitrary distribution of fibers. The impact of earthquakes on the artificial structures is of great concern to engineers and architects. During an earthquake and similar disturbances, a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend upon both ground vibration and physical properties of the structure. Most concrete structures need steel reinforcing to some extent. The study of plane and surface wave propagation in thermally conducting fiber-reinforced composites has applications in civil engineering and geophysics.

Chadwick and Seet [4] and Singh and Sharma [5] have discussed the propagation of plane harmonic waves in anisotropic thermoelastic materials. Singh [6] studied a problem on wave propagation in an anisotropic generalized thermoelastic solid and obtained a cubic equation, which gives the dimensional velocities of various plane waves. Recently, a number of investigations [7–12] have been carried out using the aforesaid theories of generalized thermoelasticity under the effect of a magnetic field.

Fiber-reinforced composites are used in a variety of structures due to their low weight and high strength. The analysis of stress and deformation of fiber-reinforced composite materials has been an important subject of solid mechanics for the last three decades. Spencer [13], Pipkin [14], and Rogers [15,16] did pioneering works on the subject. Sengupta and Nath [17] discussed the problem of surface waves in fiber-reinforced anisotropic elastic media. Recently, Singh and Singh [18] discussed the reflection of plane waves at the free surface of a fiber-reinforced elastic half-space.

Fiber-reinforced composites are widely used in engineering structures. A continuum model is used to explain the mechanical properties of such materials. Fibers are assumed as an inherent material property, rather than some form of inclusion in such models [13]. In the case of an elastic solid reinforced by a series of parallel fibers, it is usual to assume transverse isotropy. In the linear case, the associated constitutive relations, and the related infinitesimal stress and strain components, have five material constants as in Abbas [19] and Abbas and Othman [20].

In the present work, the (LS) theory is applied to study the influence of a magnetic field, time, and reinforcement on the total deformation of a body and the interactions with each other. The problem has been solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement, the stress components, and the induced magnetic field are represented graphically.

2 Formulation of the Problem

We consider the problem of a thermoelastic half-space ($x \geq 0$). A magnetic field with a constant intensity $H = (0, 0, H_0)$ acts parallel to the boundary plane (taken as the direction of the z-axis). The surface of the half-space is subjected to a thermal shock which is a function of y and t. Thus, all the quantities considered will be functions of the time variable t, and of the coordinates x and y. We begin our consideration with linearized equations of electro-dynamics of a slowly moving medium [7]:

$$\mathbf{J} = \operatorname{curl}\mathbf{h} - \varepsilon_0\dot{\mathbf{E}}, \tag{1}$$

$$\operatorname{curl}\mathbf{E} = -\mu_0\dot{\mathbf{h}}, \tag{2}$$

$$\mathbf{E} = -\mu_0(\dot{\mathbf{u}} \times \mathbf{H}), \tag{3}$$

$$\nabla \cdot \mathbf{h} = 0. \tag{4}$$

These equations are supplemented by the displacement equations of the theory of elasticity, taking into consideration the Lorentz force, to give

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i. \tag{5}$$

$$F_i = \mu_0(J \times H)_i, \tag{6}$$

The constitutive equation for a fiber-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector **a** is (Belfield et al. [21])

$$\begin{aligned}\sigma_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_\mathrm{L} - \mu_T)(a_i a_k e_{kj} \\ &\quad + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\delta_{ij}, \quad i, j, k, m = 1, 2, 3,\end{aligned} \tag{7}$$

The heat conduction equation is

$$K_{ij}T_{,ij} = \rho c_e\left(\dot{T} + t_0\ddot{T}\right) + T_0\beta_{ij}\left(\dot{u}_{i,j} + t_0\ddot{u}_{i,j}\right), i, j = 1, 2, 3. \tag{8}$$

where μ_0 is the magnetic permeability; ε_0 is the electric permeability; $\dot{u}$ is the particle velocity of the medium; **h** is the induced magnetic field vector; **E** is the induced electric field vector, **J** is the current density vector; ρ is the mass density; u_i is the displacement vector components; e_{ij} is the strain tensor; σ_{ij} is the stress tensor; T is the temperature change of a material particle; T_0 is the reference uniform temperature of the body; β_{ij} is the thermal elastic coupling tensor; c_e is the specific heat at constant strain; K_{ij} is the thermal conductivity; t_0 is the relaxation time; δ_{ij} is the Kronecker delta; λ, μ_T are elastic parameters; α, β, $(\mu_\mathrm{L} - \mu_T)$ are reinforced anisotropic elastic parameters, and $a \equiv (a_1, a_2, a_3)$, and $a_1^2 + a_2^2 + a_3^2 = 1$. The comma notation is used for spatial derivatives, and the superimposed dot represents time differentiation.

We consider the problem of a fiber-reinforced anisotropic half-space ($x \geq 0$). All the considered functions will be depend on the time t and the coordinates x and y. Thus, the displacement vector u_i will have the components,

$$u = u_x = u(x, y, t), \quad v = u_y = v(x, y, t), \quad w = u_z = 0. \tag{9}$$

We choose the fiber direction as $a \equiv (1, 0, 0)$ so that the preferred direction is the x-axis, Eqs. 5–7 simplify, as given below,

$$\sigma_{xx} = (\lambda + 2\alpha + 4\mu_\mathrm{L} - 2\mu_T + \beta)\frac{\partial u}{\partial x} + (\lambda + \alpha)\frac{\partial v}{\partial y} - \beta_{11}(T - T_0), \tag{10}$$